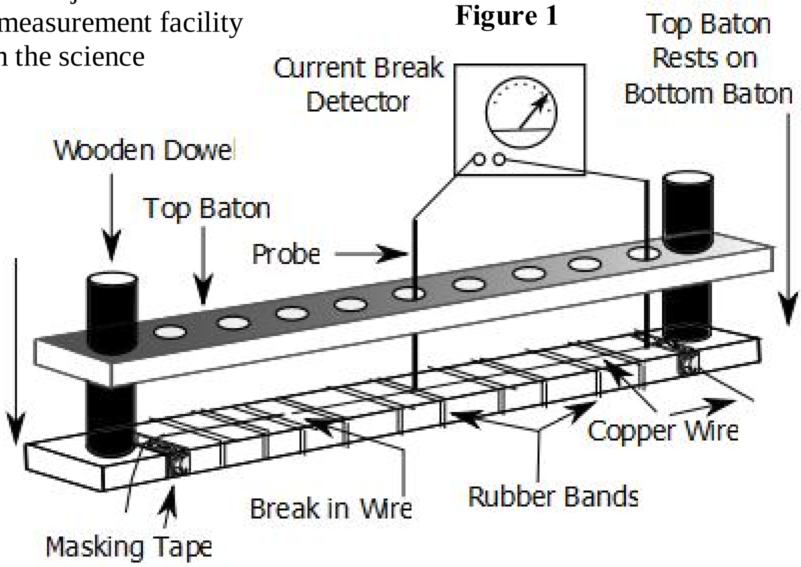
Size Does Matter Part Four - O(Log_2 n) Complexity

An example of *log_2 n* complexity can be demonstrated without going near a computer or writing computer programs or tracing pseudocode. Suppose an electrician has installed low voltage garden lights around your garden. The electrician has buried the cabling for the lights in the ground. You find that a particular light isn't getting current, even though the low voltage power supply located in the garden shed is working fine. There must be a break somewhere in the 16 metres of cable buried in the ground between the shed and the garden light at the bottom of the garden. The fault must be located without digging up too much earth. How is this possible? Well, there is a technique called the *split-half method* which can quickly locate a fault in an electrical circuit. You make a hole halfway between the shed and the garden light and test the wire at this point for current[1]. If there is current between the shed and the hole, then the break must be between the hole and the garden light. Applying the technique again, you make a second hole, half-way between the first hole and the garden light. If, on the other hand, the first test registered no current, then the break must be between the hole and the shed. In this case, you would make the second hole halfway between the first hole and the shed. In this way the number of holes that must be made to find the break is minimized and so also the time taken.

Figure 1 shows how this might be simulated in the classroom. Two batons slot together on wooden dowels to cover a length of bare copper wire strapped to the lower baton with two lengths of sticky masking tape and several elastic bands. The copper wire is cut and separated slightly as shown in the diagram so that the small gap is invisible through the holes in the top baton. These holes are spaced evenly along the length of the baton with the two opposite end holes approximately 64 cm apart. Exactly 32 cm from each of the end holes is another hole at the centre of the top baton. Sections of the copper wire are probed and tested with the current break detector as shown ï the full length, then one half then one quarter and so on. The current break detector is any circuit continuity tester. Do-it-yourself and electronic stores sell multi-meters for about £6 to £8 that do the job.
Multi-meters must have a resistance measurement facility (unit q) or one can be borrowed from the science department at your school/college.

A simpler version can be constructed with just one baton, masking tape, rubber bands and copper wire. A strip of masking tape with holes made at fixed intervals covers the length of copper wire. The baton is marked with intervals labelled, 0, 8, 16, 24, 32, 40, 48, 56, 64.

Figure 1

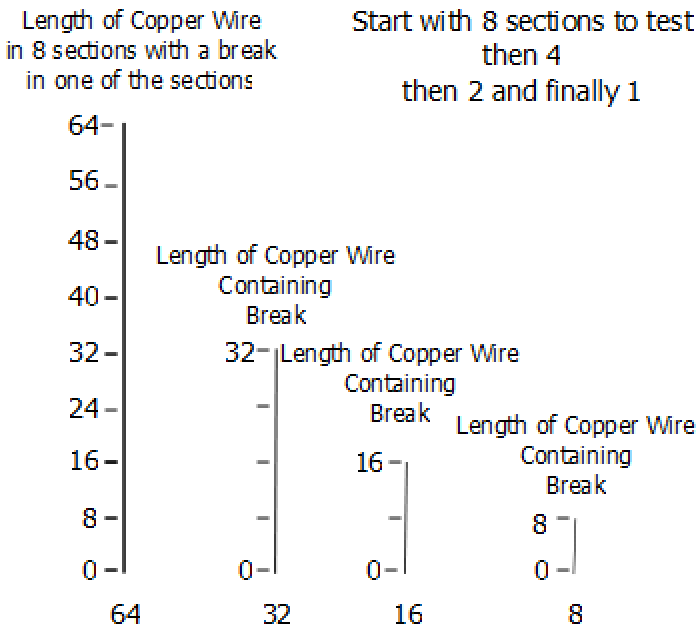


With the probes of the current break detector in holes 0cm and 64cm the instrument should indicate a high resistance to current flow. Now move the 64cm probe to the 32cm hole. If the meter reading is still high then the break is in the section between 0cm and 32cm. If the meter reading is zero or very low then the break is in the section between 32cm and 64cm. Let's say that the break is between 0cm and 8cm. Then the meter should indicate a high resistance to current flow when a measurement is made with the probes between holes at 0cm and 64cm and at 0cm and 32cm respectively. Next place the probes in 0cm and 16cm where a high

[1] Using a special purpose wire piercing tool

reading should once again be obtained. Finally, test the last section 0cm and 8cm. The following lengths of copper wire will be tested: 0-64, 0-32, 0-16 and 0-8 if the break is in section 0-8. Since we have been told that there is a break somewhere between 0cm and 64cm we don't need to include this test in the count. This leaves a count of 3 for the number of tests. There are 8 sections in total but the split-half technique has reduced the number of tests to 3. The same answer would be obtained if the break was in any of the other sections, e.g. 8-16. Try this for yourself. Pick a section at random for the break section and then count the number of tests to find this section.

Figure 2



How many tests would need to be performed to test for a single break among 16 sections between 0cm and 4cm?

Answer = 4 because the following sections will be tested
0-32
0-16
0-8
0-4

How many tests would need to be performed to test for a single break among 32 sections between 0cm and 2cm?

Answer = 5 because the following sections will be tested
0-32
0-16
0-8
0-4
0-2

How many tests would need to be performed to test for a single break among 64 sections between 0cm and 1cm?

Answer = 6 because the following sections will be tested
0-32
0-16
0-8
0-4
0-2
0-1

The number of tests increases by one when the number of sections is doubled. Try breaks in different sections for each of the above cases. The same results for the number of tests should be obtained. Just rearranging the graphics in Figure 2 should be enough to convince you.

Figure 3 shows that the number of tests is 4 when the number of sections is 16, 5 when the number of sections is 32, 6 when the number of sections is 64 and so on. The number of tests is the same as the exponent when the number of sections is expressed as a power of 2, e.g. 16 expressed as a power of 2 is 2^4. The exponent in this example is 4.

Figure 3

No Of Sections	$2^{\text{No Of Sections}}$	No Of Tests
128	2^7	7
64	2^6	6
32	2^5	5
16	2^4	4
8	2^3	3
4	2^2	2
2	2^1	1

Conclusion:

No of Tests = $\mathbf{Log_2}$ No of Sections

Therefore, the growth rate of number of tests for this technique is $O(\log_2 n)$ where n is the number of sections. As the number of sections doubles the number of tests increases by 1 each time. This is shown in Figure 4 where the number of tests grows by one each time the number of sections is doubled.

Mathematically,

No of Sections = 2^k

Log_2 No of Sections = $\log_2 2^k$ Taking logs to the base 2 of both sides,

Therefore, Log_2 No of sections = k because $\log_2 2^k = k$

Figure 4 is a plot of **$\mathbf{Log_2}$ No of Sections** against **No of Tests** using a logarithmic scale for the **No of Sections** axis, i.e. exponents of $\mathbf{2^{\text{No of Sections}}}$ are used to mark off this axis at equal intervals.

Figure 4